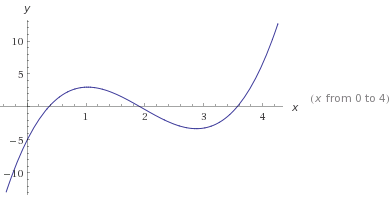
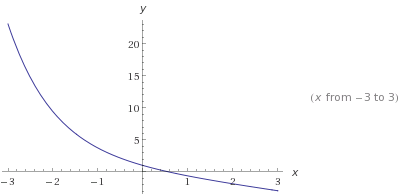
**F(x) = 2x3 - 11.7x2 + 17.7x - 5**



As we can see in this graph, the points that are to be bracketed to are (0,1), (1,2),(2,4).

**F(x) = e-x - x**



**Approximate Error Tables**

**Part A, Root 1)**

**Bisection Method**

Converges at c = 0.3652

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| n | a | b | c | f(a) | f(b) | f(c) | Error |
| 0 | 0.0 | 1.0 | 0.5 | -5.0 | 3.0 | 1.1750 | None |
| 1 | 0.0 | 0.5 | 0.25 | -5.0 | 1.1750 | -1.275 | 1.0 |
| 2 | 0.25 | 0.5 | 0.375 | -1.275 | 1.1750 | 0.0977 | 0.3333 |
| 3 | 0.25 | 0.375 | 0.3125 | -1.275 | 0.0977 | -0.5503 | 0.2 |
| 4 | 0.3125 | 0.375 | 0.3438 | -0.550 | 0.0977 | -0.2169 | 0.0909 |
| 5 | 0.3438 | 0.375 | 0.3594 | -0.2169 | 0.0977 | -0.0573 | 0.0435 |
| 6 | 0.3594 | 0.375 | 0.3672 | -0.0573 | 0.0977 | 0.0208 | 0.0213 |
| 7 | 0.3594 | 0.3672 | 0.3633 | -0.0573 | 0.0208 | -0.0181 | 0.0108 |
| 8 | 0.3633 | 0.3672 | 0.3652 | -0.0181 | 0.0208 | 0.0014 | 0.0053 |

Like stated above, looking at the graph, we can see the points when it reaches 0 are: (0,1), (1,2), (2,4) so starting point (0,1) was used as our initial guess.

**Newton-Raphson Method**

Converges at c = 0.3651

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| n | x | f(x) | f'(x) | Error |
| 0 | 0.0 | -5.0 | 17.7 | None |
| 1 | 0.2824 | -0.8886 | 11.5686 | 1.0 |
| 2 | 0.3593 | -0.0581 | 10.0671 | 0.2138 |
| 3 | 0.3651 | -3.1770 | 9.9571 | 0.0158 |

The best method of picking a number was to pick the number that was closest to the root which would output the least number of iterations.

**Secant Method**

Converges at c = 0.3652

|  |  |  |  |
| --- | --- | --- | --- |
| n | x | f(x) | Error |
| 0 | 0.0 | -5.0 | None |
| 1 | 1.0 | 3.0 | 0.7632 |
| 2 | 0.625 | 1.9805 | 0.6 |
| 3 | 0.4477 | 0.7585 | 0.3961 |
| 4 | 0.3376 | -0.2809 | 0.3260 |
| 5 | 0.3674 | 0.0224 | 0.0810 |
| 6 | 0.3652 | 5.7943 | 0.0060 |

For the initial starting point, it would be the same as the bisection and false position method in which they are closest to the root.

**False Position**

Converges at c = 0.3656

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| n | a | b | c | f(a) | f(b) | f(c) | Error |
| 0 | 0.0 | 1.0 | 0.625 | -5.0 | 3.0 | 1.980 | None |
| 1 | 0.0 | 0.625 | 0.4477 | -5.0 | 1.9805 | 0.7585 | 0.3961 |
| 2 | 0.0 | 0.4477 | 0.3887 | -5.0 | 0.7585 | 0.2298 | 0.1517 |
| 3 | 0.0 | 0.3887 | 0.3716 | -5.0 | 0.2298 | 0.0646 | 0.0460 |
| 4 | 0.0 | 0.3716 | 0.3669 | -5.0 | 0.0646 | 0.0178 | 0.0129 |
| 5 | 0.0 | 0.3669 | 0.3656 | -5.0 | 0.0178 | 0.0049 | 0.0036 |

False position has the same starting point with bisection because they have similar methods to approach the root.

**Modified Secant**

Converges at c = 0.0365

|  |  |  |  |
| --- | --- | --- | --- |
| n | x | f(x) | Error |
| 0 | 0.0 | 3.0 | None |
| 1 | 0.01 | -4622.1182 | 1.0088 |
| 2 | -0.114 | -1382.1475 | 0.6223 |
| 3 | -0.070 | -411.0295 | 0.7063 |
| 4 | -0.041 | -120.7745 | 0.7063 |
| 5 | -0.022 | -34.50467 | 1.2821 |
| 6 | -0.010 | -9.1663 | 3.6500 |
| 7 | -0.002 | -1.9704 | 2.0544 |
| 8 | 0.00196 | -0.2204 | 0.4304 |
| 9 | 0.00343 | -0.0037 | 0.0585 |
| 10 | 0.00365 | 1.14797 | 0.0010 |

If we were to choose negative values, the method would converge but to a slow extent. When the points 0 are chosen, then there would be an error. We can also see that when the values get bigger, than the iterations increases.

**Approximate Error Graph**

**Part A, Root 2)**

**Bisection Method**

Converges at c = 1.9219

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| n | a | b | c | f(a) | f(b) | f(c) | Error |
| 0 | 1.0 | 2.0 | 1.5 | 3.0 | -0.4000 | 1.9750 | None |
| 1 | 1.5 | 2.0 | 1.75 | 1.9750 | -0.4000 | 0.8625 | 0.1429 |
| 2 | 1.75 | 2.0 | 1.875 | 0.8625 | -0.4000 | 0.2383 | 0.0667 |
| 3 | 1.875 | 2.0 | 1.9375 | 0.2383 | -0.4000 | -0.0806 | 0.0323 |
| 4 | 1.875 | 1.9375 | 1.9063 | 0.2383 | -0.0806 | 0.0791 | 0.0164 |
| 5 | 1.9063 | 1.9375 | 1.9219 | 0.0791 | -0.0806 | -6.8512 | 0.0081 |

Like stated before, looking at the graph, we can see the points when it reaches 0 are: (1,2), so starting point (1,2) was used as our initial guess.

**Newton-Raphson Method**

Converges at c = 1.9219

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| n | x | f(x) | f'(x) | Error |
| 0 | 2.0 | -0.4000 | -5.1000 | None |
| 1 | 1.922 | 8.8051 | -5.1101 | 0.0408 |

**Secant Method**

Converges at c = 1.9217

|  |  |  |  |
| --- | --- | --- | --- |
| n | x | f(x) | Error |
| 0 | 1.0 | 3.0 | None |
| 1 | 2.0 | -0.4000 | 2.5264 |
| 2 | 1.8824 | 0.2009 | 0.0625 |
| 3 | 1.9457 | -0.1224 | 0.0325 |
| 4 | 1.9217 | 1.8920 | 0.0125 |
| 5 | 1.9217 | 1.0785 | 1.9255 |

**False Position**

Converges at c = 1.9217

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| n | a | b | c | f(a) | f(b) | f(c) | Error |
| 0 | 1.0 | 2.0 | 1.8824 | 3.0 | -0.4000 | 0.2009 | None |
| 1 | 1.8824 | 2.0 | 1.9217 | 0.2009 | -0.4000 | 2.8315 | 0.0205 |
| 2 | 1.9217 | 2.0 | 1.9217 | 2.8315 | -0.4000 | 5.6998 | 2.8827 |

**Modified Secant**

Converges at c = 1.9217

|  |  |  |  |
| --- | --- | --- | --- |
| n | x | f(x) | Error |
| 0 | 0.0 | -0.4000 | None |
| 1 | 0.02 | 0.0014 | 0.0408 |
| 2 | 0.0192 | 6.9337 | 1.4407 |

**Part A, Root 3)**

**Bisection Method**

Converges at c = 3.5938

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| n | a | b | c | f(a) | f(b) | f(c) | Error |
| 0 | 2.0 | 4.0 | 3.0 | -0.4000 | 6.6000 | -3.2000 | None |
| 1 | 3.0 | 4.0 | 3.5 | -3.2000 | 6.6000 | -0.6250 | 0.1429 |
| 2 | 3.5 | 4.0 | 3.75 | -0.6250 | 6.6000 | 2.3125 | 0.0667 |
| 3 | 3.5 | 3.75 | 3.625 | -0.6250 | 2.3125 | 0.6867 | 0.0345 |
| 4 | 3.5 | 3.625 | 3.563 | -0.6250 | 0.6867 | -0.0069 | 0.0175 |
| 5 | 3.5625 | 3.625 | 3.5938 | -0.0069 | 0.6867 | 0.3303 | 0.0087 |

**Newton-Raphson Method**

Converges at c = 3.5632

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| n | x | f(x) | f'(x) | Error |
| 0 | 4.0 | 6.6000 | 20.1000 | None |
| 1 | 3.6716 | 1.2554 | 12.6693 | 0.0894 |
| 2 | 3.5726 | 0.0995 | 10.6811 | 0.0277 |

**Secant Method**

Converges at c = 1.9217

|  |  |  |  |
| --- | --- | --- | --- |
| n | x | f(x) | Error |
| 0 | 2.0 | -0.4000 | None |
| 1 | 4.0 | 6.6000 | 6.0529 |
| 2 | 2.1143 | -0.9760 | 0.8919 |
| 3 | 1.9206 | 0.0057 | 0.0413 |
| 4 | 1.9217 | -4.6062 | 5.8356 |

**False Position**

Converges at c = 3.5473

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| n | a | b | c | f(a) | f(b) | f(c) | Error |
| 0 | 2.0 | 4.0 | 2.1143 | -0.4000 | 6.6000 | -0.9760 | None |
| 1 | 2.1143 | 4.0 | 2.3572 | -0.9760 | 6.6000 | -3.2158 | 0.1437 |
| 2 | 2.3572 | 4.0 | 2.7526 | -2.0923 | 6.6000 | -3.2158 | 0.1437 |
| 3 | 2.7526 | 4.0 | 3.1613 | -3.2158 | 6.6000 | -2.7857 | 0.1293 |
| 4 | 3.1613 | 4.0 | 3.4102 | -2.7857 | 6.6000 | -1.3864 | 0.0730 |
| 5 | 3.4102 | 4.0 | 3.5126 | -1.3864 | 6.6000 | -0.5063 | 0.0291 |
| 6 | 3.5126 | 4.0 | 3.5473 | -0.5063 | 6.6000 | -0.1638 | 0.0098 |

**Modified Secant**

Converges at c = 3.5638

|  |  |  |  |
| --- | --- | --- | --- |
| n | x | f(x) | Error |
| 0 | 0.0 | 6.6000 | None |
| 1 | 0.04 | 1.3560 | 0.08709 |
| 2 | 0.0368 | 0.1465 | 0.02868 |
| 3 | 0.0358 | 0.0063 | 0.00370 |

**Part B, Root True)**

**Bisection Method**

Converges at c = 0.5625

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| n | a | b | c | f(a) | f(b) | f(c) | Error |
| 0 | 0.0 | 1.0 | 0.5 | 1.0 | -0.6321 | 0.1065 | 0.1184 |
| 1 | 0.5 | 1.0 | 0.75 | 0.1065 | -0.6321 | -0.2776 | 0.3333 |
| 2 | 0.5 | 0.75 | 0.625 | 0.1065 | -0.2776 | -0.0897 | 0.1020 |
| 3 | 0.5 | 0.625 | 0.5625 | 0.1065 | -0.0897 | 0.0073 | 0.0082 |

**Newton-Raphson Method**

Converges at c = 0.0

**Secant Method**

Converges at c = 0.5722

|  |  |  |  |
| --- | --- | --- | --- |
| n | x | f(x) | Error |
| 0 | 0.0 | 1.0 | 1.0 |
| 1 | 1.0 | -0.6321 | 0.7632 |
| 2 | 0.6127 | -0.0708 | 0.0803 |
| 3 | 0.5722 | -0.0079 | 0.0089 |

**False Position**

Converges at c = 0.5722

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| n | a | b | c | f(a) | f(b) | f(c) | Error |
| 0 | 0.0 | 1.0 | 0.6127 | 1.0 | -0.6321 | -0.0708 | 0.0803 |
| 1 | 0.0 | 0.6127 | 0.5722 | 1.0 | -0.0708 | -0.0079 | 0.0089 |

**Modified Secant**

Converges at c = 0.5670

|  |  |  |  |
| --- | --- | --- | --- |
| n | x | f(x) | Error |
| 0 | 0.0 | -0.6321 | 0.7632 |
| 1 | 0.01 | 0.0471 | 0.0527 |
| 2 | 0.0054 | 2.0938 | 2.3557 |

**Part B, Root Approx)**

**Bisection Method**

Converges at c = 0.5664

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| n | a | b | c | f(a) | f(b) | f(c) | Error |
| 0 | 0.0 | 1.0 | 0.5 | 1.0 | -0.6321 | 0.1065 | None |
| 1 | 0.5 | 1.0 | 0.75 | 0.1065 | -0.6321 | -0.2776 | 0.3333 |
| 2 | 0.5 | 0.75 | 0.625 | 0.1065 | -0.2776 | -0.0897 | 0.2 |
| 3 | 0.5 | 0.625 | 0.5625 | 0.1065 | -0.0897 | 0.0073 | 0.1111 |
| 4 | 0.5625 | 0.625 | 0.5938 | 0.0073 | -0.0897 | -0.0415 | 0.0526 |
| 5 | 0.5625 | 0.5938 | 0.5781 | 0.0073 | -0.0415 | -0.0172 | 0.0270 |
| 6 | 0.5625 | 0.5781 | 0.5703 | 0.0073 | -0.0172 | -0.0050 | 0.0137 |
| 7 | 0.5625 | 0.5703 | 0.5664 | 0.0073 | -0.0050 | 0.0012 | 0.0069 |

**Newton-Raphson Method**

Converges at c = 0.5671

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| n | x | f(x) | f'(x) | Error |
| 0 | 0.0 | 1.0 | -2.0 | None |
| 1 | 0.5 | 0.1065 | -1.6065 | 1.0 |
| 2 | 0.5663 | 0.0013 | -1.5676 | 0.1171 |

**Secant Method**

Converges at c = 0.5671

|  |  |  |  |
| --- | --- | --- | --- |
| n | x | f(x) | Error |
| 0 | 0.0 | 1.0 | None |
| 1 | 1.0 | -06321 | 0.7632 |
| 2 | 0.6127 | -0.0708 | 0.6321 |
| 3 | 0.5722 | -0.0079 | 0.0708 |
| 4 | 0.5671 | 6.4583 | 0.0090 |

**False Position**

Converges at c = 0.5677

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| n | a | b | c | f(a) | f(b) | f(c) | Error |
| 0 | 0.0 | 1.0 | 0.6127 | 1.0 | -0.6321 | -0.0708 | None |
| 1 | 0.0 | 0.6127 | 0.5722 | 1.0 | -0.0708 | -0.0079 | 0.0708 |
| 2 | 0.0 | 0.5722 | 0.5677 | 1.0 | -0.0079 | -8.7739 | 0.0079 |

**Modified Secant**

Converges at c = 0.5671

|  |  |  |  |
| --- | --- | --- | --- |
| n | x | f(x) | Error |
| 0 | 0.0 | -0.6321 | None |
| 1 | 0.01 | 0.0471 | 0.8613 |
| 2 | 0.0054 | 2.0938 | 0.0525 |
| 3 | 0.0057 | -2.0959 | 2.3581 |